Welcome to my Statistics Review

We will begin at 1pm! - Kyle

Review Agenda

- Brief comment on hypothesis tests
- Connect hypothesis tests & confidence intervals
- Interpret p-value to make conclusion
- Word problem → Calculator → Conclusion
- Outline of each hypothesis test
- Practice
- Comment on confidence intervals

On Hypothesis Tests…

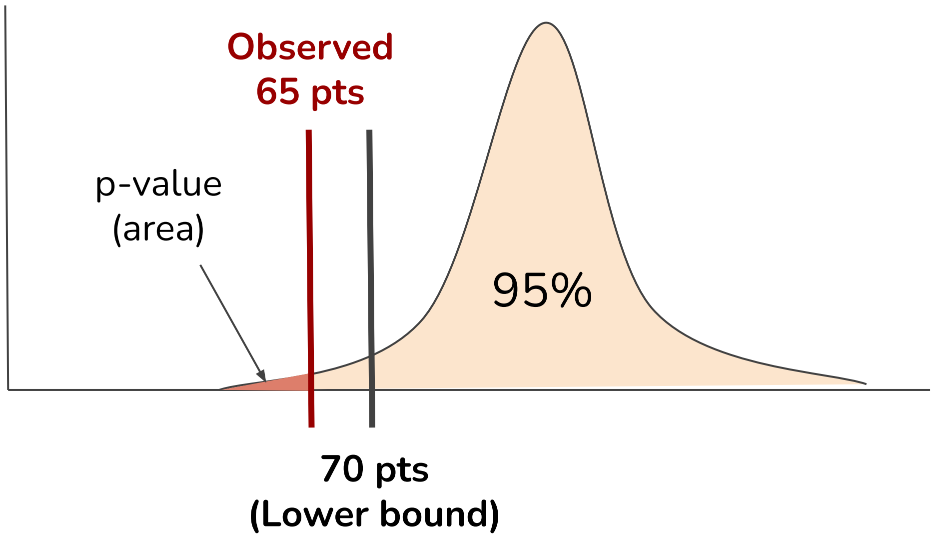
- One big proof by contradiction
- Imagine…
  - You wake up, and wonder if it rained last night
  - You are ***95% confident*** that if it rained last night, you would see rain puddles on the ground
  - You make an **observation**, and see no rain puddles
  - This is a contradiction! So you reject the “null” and claim there is sufficient evidence to suggest that it didn’t rain

A Numerical Example...

- Now imagine...
  - You want to know if the ASU basketball team won their game
  - You are ***95% confident*** that if they won, they would score *at least* 70 points
  - You **observe** that they scored 65 points
  - This is a contradiction! So you reject the “null” and claim there is sufficient evidence to suggest that they didn’t win

A Numerical Example...

Point Distribution of Won Games

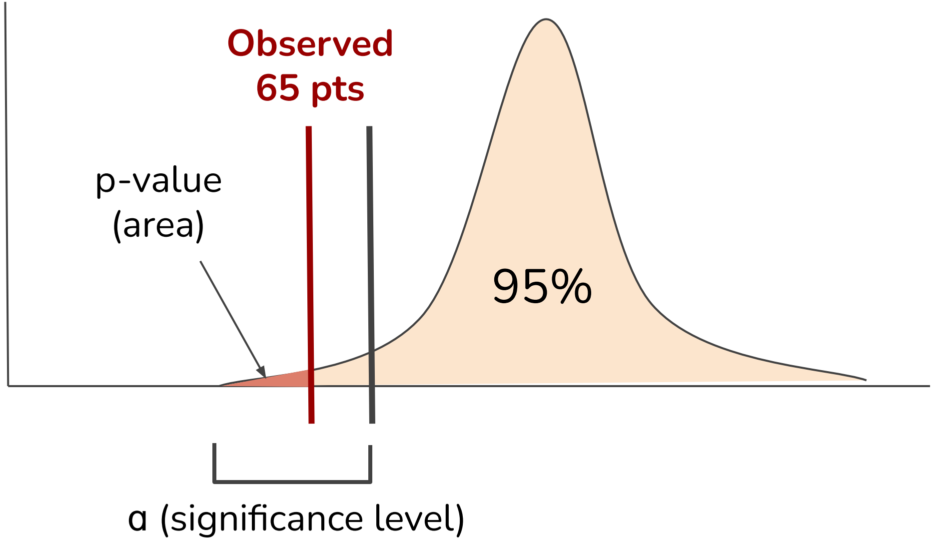


P-Value:

The probability that you were to have observed the value that you did, or a more “extreme” value

A Numerical Example...

Point Distribution of Won Games



P-Value:

The probability that you were to have observed the value that you did, or a more “extreme” value

General Outline of a Solution

- Given: bunch of words and numbers
  - Interpret & extract relevant values (NEXT)
  - Choose the *appropriate* hypothesis test (SOON)
  - Plug stuff into calculator (LATER)
    - Some exceptions unfortunately :(
  - Use p-value to make conclusion (LATER)

Interpret & Extract Values

Statistics:

- Sample mean $\overline{x}_1, \overline{x}_2$
- Population standard deviation σ_1, σ_2
- Sample standard deviation s_1, s_2
- Sample size n_1, n_2
- Sample proportion $\hat{p}_1, \hat{p}_2$
- Significance level α

Interpret & Extract Values

To Do List:

1. Determine 1-Sample or 2-Sample (Easy)
2. Identify the values:
    a. Easy: $\overline{x}, \hat{p}, n, \alpha$
    b. Harder: σ vs. s
        i. *Rule of Thumb:* If std. deviation is mentioned *before* sample mean, it's probably the population std. deviation. If std. deviation is mentioned *with* sample mean, it's probably the sample std. deviation. If there's no mention, just *raw data*, then it's definitely sample std. deviation.

Interpret & Extract Values

Example 1:

Two different formulas of an oxygenated motor fuel are being tested to study their road octane numbers. The variance of road octane number for formula 1 is 1.5, and for formula 2 it is 1.2. Two random samples of size 15 and 20 are tested, and the mean octane numbers observed are 88.9 fluid ounces and 92.3 fluid ounces. Assume normality.

Interpret & Extract Values

2-Sample

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Interpret & Extract Values

Example 1:

2-Sample

Sample sizes

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Sample means

Interpret & Extract Values

Example 1:

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2-Sample

Population variances

Sample sizes

Sample means

Interpret & Extract Values

Example 2:

Raw Data → Sample std. deviation!!!

Two catalysts are being analyzed to determine how they affect the mean yield of a chemical process. Specifically, catalyst 1 is currently in use, but catalyst 2 is acceptable. Since catalyst 2 is cheaper, it should be adopted, providing it does not change the process yield. A test is run in the pilot plant and results in the data shown below. Is there any difference between the mean yields? Use $\alpha = 0.01$.

Observation Number	Catalyst 1	Catalyst 2
1	91.50	89.19
2	94.18	90.95
3	92.18	90.46
4	95.39	93.21

Interpret & Extract Values

To Do List (Continued):

- Determine the statistic being tested (easier)
  - Ask "*what is the purpose of this word problem?*"
  - *Possible statistics:* will be reviewed later
- Determine the alternative hypothesis (a little harder)
  - (For 1 sample) : *What are we comparing this statistic to?*
    - μ_0, σ_0, p_0
  - (>) keywords : higher, larger, greater, more, etc.
  - (<) keywords : smaller, shorter, less, etc.
  - (≠) keywords : *different*

Interpret & Extract Values

Summary:

- We have now found four important parameters of the problems:
  - Type of test (1 or 2 sample)
  - The observed statistics
  - The statistic being tested
  - The alternative hypothesis
    - Note that we didn't have to find the null hypothesis because each statistic we test has one and only one null hypothesis (in this class)

Choose the Appropriate Test

Based on the test type & observed statistics:

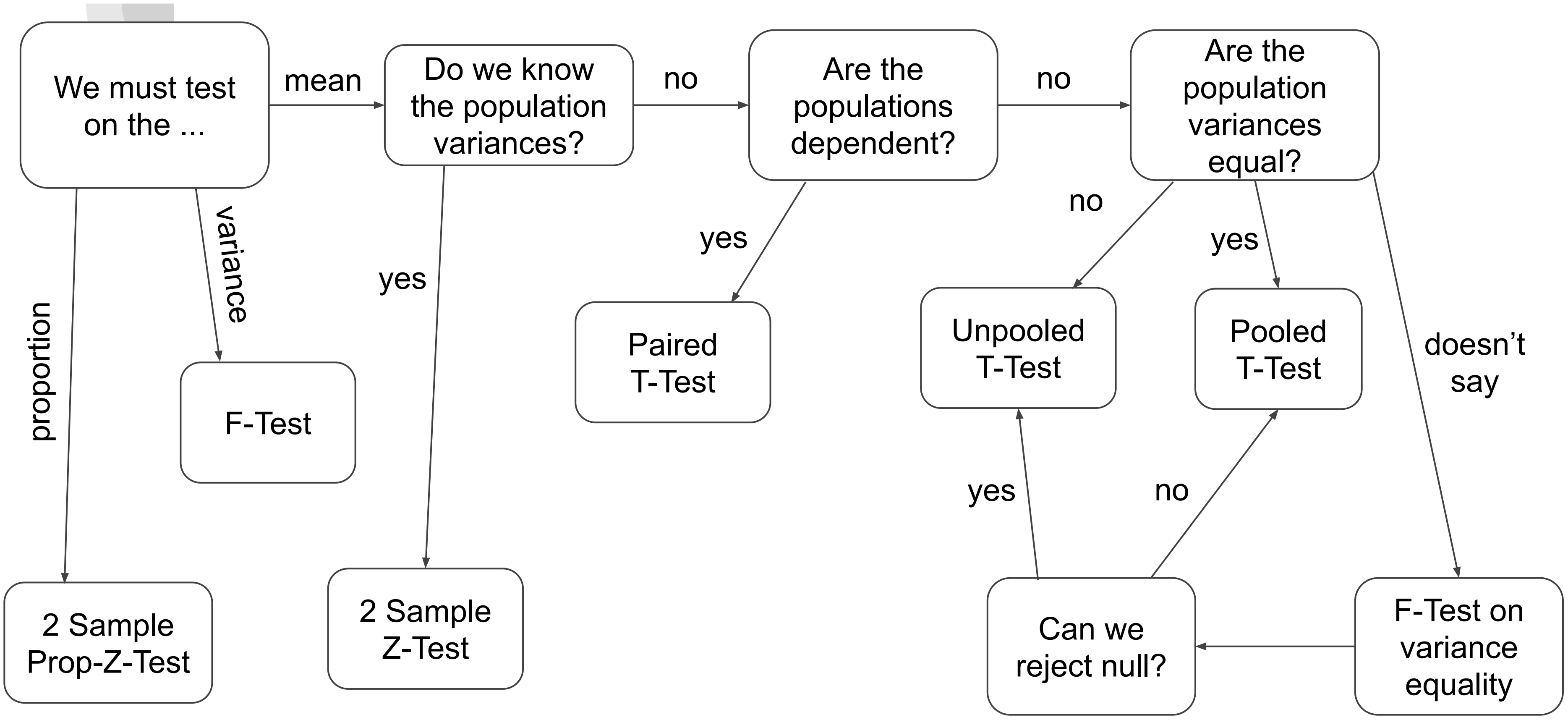
1-Sample Test	$\overline{x}$	σ	s	n	$\hat{p}$	α
Z-Test (on mean, population variance known)	yes	yes	no	yes	no	yes
T-Test (on mean, population variance not known)	yes	no	yes	yes	no	yes
Chi-Squared Test (on variance)	no	no	yes	yes	no	yes
Prop-Z Test (on proportion)	no	no	no	yes	yes	yes

Choose the Appropriate Test

Based on the test type & observed statistics:

2-Sample Test	$\overline{x}$	σ	s	n	$\hat{p}$	α
Z-Test	yes	yes	no	yes	no	yes
F-Test	no	no	yes	yes	no	yes
Pooled T-Test	yes	no	yes	yes	no	yes
Unpooled T-Test	yes	no	yes	yes	no	yes
Paired T-Test	yes	no	yes	yes	no	yes
Prop-Z Test	no	no	no	yes	yes	yes

Choose the Appropriate Test (Based on context)



Calculator Input (TI 83/84 used)

Three steps:

1. Find function in calculator
2. Input observed statistics & alternative hypothesis
3. Receive two outputs
   a. Test statistic
   b. P-value

1-Sample Test	Function
Z-Test (on mean, population variance known)	STAT > TESTS > 1: Z-Test
T-Test (on mean, population variance not known)	STAT > TESTS > 2: T-Test
Chi-Squared Test (on variance)	None :(
Prop-Z Test (on proportion)	STAT > TESTS > 5: 1-PropZTest

Calculator Input

Three steps:

1. Find function in calculator
2. Input observed statistics & alternative hypothesis
3. Receive two outputs
   a. Test statistic
   b. P-value

2-Sample Test	Function
Z-Test	STAT > TESTS > 3: 2-SampZTest
F-Test	STAT > TESTS > E: 2-SampFTest
Pooled T-Test	STAT > TESTS > 4: 2-SampTTest
Unpooled T-Test	STAT > TESTS > 4: 2-SampTTest
Paired T-Test	STAT > TESTS > 2: T-Test
Prop-Z Test	STAT > TESTS > 6: 2-PropZTest

Interpret P-Value

Overview:

- If p-value < ɑ (significance level)
  - Reject the null hypothesis (NOT accept the alternative)
  - “There is sufficient evidence to suggest that [INSERT ALT HYPOTHESIS]”
- If p-value ≥ ɑ (significance level)
  - Cannot reject the null hypothesis (NOT accept the null)
  - “There is insufficient evidence to suggest that [INSERT ALT HYPOTHESIS]”

Practice Example 1

According the the U.S. Department of Education, full-time graduate students receive an average salary of $12,800. The dean of graduate studies at a large state university in PA claims that his graduate students earn more than this. He surveys 46 randomly selected students and finds their average salary is $13,445 with a standard deviation of $1800. With alpha = 0.05, is the dean's claim correct?

Solution
p-value: 0.0096
p < alpha, so reject null

Practice Example 1

μ_0

n

$>$

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s

α

$\bar{x}$

Practice Example 2

Two college instructors are interested in whether or not there is any variation in the way they grade math exams. They each grade the same set of 10 exams. The first instructor’s grades have a variance of 52.3. The second instructor’s grades have a variance of 89.9. Test the claim that the first instructor’s variance is smaller. (In most colleges, it is desirable for the variances of exam grades to be nearly the same among instructors.) The level of significance is 10%.

Solution
p-value: 0.216
p > alpha, so can't reject null

Practice Example 2

s_1^2, s_2^2

n_1, n_2

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α

$<$

Practice Example 3

Researchers wanted to see with 0.01 significance if people who exercise regularly sleep better than people who don't. They took a random sample of adult males and surveyed them about their exercise routine and their sleep duration. Here is a summary of the results:

	Exercise	No exercise
Mean	7 hours	6.5 hours
Standard deviation	1 hour	1.5 hours
Number of people	180	190

What would be an appropriate test statistic for the researchers' test?

Practice Example 3

α

Solution
P-value: 8.92 x 10^-5
p < alpha, so reject null

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$>$

	Exercise	No exercise	
Mean	7 hours	6.5 hours	$\bar{x}_1, \bar{x}_2$
Standard deviation	1 hour	1.5 hours	s_1, s_2
Number of people	180	190	n_1, n_2

What would be an appropriate test statistic for the researchers' test?

Practice Example 4

Suppose a company develops a new drug designed to prevent colds. The company states that the drug is equally effective for men and women. To test this claim, they choose a simple random sample of 100 women and 200 men from a population of 100,000 volunteers. At the end of the study, 38% of the women caught a cold; and 51% of the men caught a cold. Based on these findings, can we reject the company's claim that the drug is equally effective for men and women? Use a 0.05 level of significance.

Solution
P-value: 0.033
p < alpha, so reject null

Practice Example 4

n_1, n_2

Suppose a company develops a new drug designed to prevent colds. The company states that the drug is equally effective for men and women. To test this claim, they choose a simple random sample of 100 women and 200 men from a population of 100,000 volunteers. At the end of the study, 38% of the women caught a cold; and 51% of the men caught a cold. Based on these findings, can we reject the company's claim that the drug is equally effective for men and women? Use a 0.05 level of significance.

$\neq$

$\hat{p}_1, \hat{p}_2$

α

Practice Example (Paired T-Test)

Twelve cars were equipped with radial tires and driven over a test course. Then the same 12 cars (with the same drivers) were equipped with regular belted tires and driven over the same course. After each run, the cars' gas economy (in km/l) was measured. Is there evidence that radial tires produce better fuel economy? (Assume normality of data, and use alpha = .05.)

Gas eco.	1	2	3	4	5	6	7	8	9	10	11	12
Radial	4.2	4.7	6.6	7.0	6.7	4.5	5.7	6.0	7.4	4.9	6.1	5.2
Belted	4.1	4.9	6.2	6.9	6.8	4.4	5.7	5.8	6.9	4.7	6.0	4.9

Practice Example (Paired T-Test)

Solution
P-value: 0.015
p < alpha,
so reject null

Twelve cars were equipped with radial tires and driven over a test course. Then the same 12 cars (with the same drivers) were equipped with regular belted tires and driven over the same course. After each run, the cars' gas economy (in km/l) was measured. Is there evidence that radial tires produce better fuel economy? (Assume normality of data, and use alpha = .05.) ← α

Gas eco.	1	2	3	4	5	6	7	8	9	10	11	12
Radial	4.2	4.7	6.6	7.0	6.7	4.5	5.7	6.0	7.4	4.9	6.1	5.2
Belted	4.1	4.9	6.2	6.9	6.8	4.4	5.7	5.8	6.9	4.7	6.0	4.9
Difference	0.1	-0.2	0.4	0.1	-0.1	0.1	0	0.2	0.5	0.2	0.1	0.3

On Confidence Intervals...

Overview:

- Same general steps taken
  - Extract relevant values (observed statistics)
  - Determine the statistic used to make the confidence interval
  - Choose appropriate confidence interval
    - If two-sided: use calculator
    - If one-sided: must use equations from lecture

On Confidence Intervals...

Two-sided calculator functions:

Description	Statistic	Distribution	Function
Population mean (variance known)	μ	Normal	STAT > TESTS > 7: ZInterval
Population mean (variance not known)	μ	T	STAT > TESTS > 8: TInterval
Population variance	σ^2	Chi-Squared	None :(
Population proportion	p	Normal	STAT > TESTS > A: 1-PropZInt

Description	Statistic	Distribution	Function
Difference in means (variances known)	$\mu_1 - \mu_2$	Normal	STAT > TESTS > 9: 2-SampZInt
Difference in variance	σ_1^2/σ_2^2	F	None :(
Difference in means (variances unknown, but equal)	$\mu_1 - \mu_2$	T	STAT > TESTS > 0: 2-SampTInt (Pooled? Yes)
Difference in means (variances unknown, but not equal)	$\mu_1 - \mu_2$	T	STAT > TESTS > 0: 2-SampTInt (Pooled? No)
Mean difference in paired observations	μ_d	T	STAT > TESTS > 8: TInterval
Difference in proportions	$p_1 - p_2$	Normal	STAT > TESTS > B: 2-PropZInt

Also Make Sure to Cover...

- Type I vs. Type II errors
  - alpha, beta, power (1 - beta)
- Chi Squared Test & CI, and F Test CI
  - Write down equations!!!
- One-sided confidence interval
  - Write down equations!!!
- Sample size needed to produce given CI
  - Write down equations!!!

One more topic to cover...

Calculate z_x **(ex.** z_α **or** $z_{\alpha/2}$ **)**

1. Go to function: DISTR > 3: InvNorm
2. Input the following values:
   a. area = *x* , μ = 0, σ = 1
   b. Tail = LEFT
3. Click Paste, and take the **absolute value** of the output

One more topic to cover...

Calculate $t_{x,y}$ **(ex.** $t_{\alpha/2,n-1}$ **)**

1. Go to function: DISTR > 4: InvT
2. Input the following values:
   a. area = *x* , df = *y*
3. Click Paste, and take the **absolute value** of the output

One more topic to cover...

Calculate $\chi^2_{x,y}$ **(ex.** $\chi^2_{\alpha/2,n-1}$ **or** $\chi^2_{1-\alpha/2,n-1}$ **)**

**Assuming you have a program (let's call it INVCHI)

1. Go to function: PRGM > EXEC > INVCHI
2. Input the following values:
   a. area (or A) = *x* , DF = *y*
3. The output is the correct value

One more topic to cover...

Calculate $f_{x,y,z}$ **(ex.** $f_{\alpha/2,n_2-1,n_2-1}$ **)**

**Assuming you have a program (let's call it INVF)

1. Go to function: PRGM > EXEC > INVF
2. Input the following values:
   a. area (or A) = *x* , DF1 = *y* , DF2 = z
3. The output is the correct value

Last Tips...

- Open (printed) notes, so print anything that might be useful
    - Also don't wait last minute to print everything. It's stressful
- Make sure to have the following programs in your calculator:
    - Chi squared inverse (find on Canvas)
    - F inverse (find on Canvas)
- If you have time to double check, try to use a different method to come to the same answer (decreases chance of silly error)
- Don't let yourself get stuck on a problem; time may be of the essence, so feel free to skip around

GOOD LUCK!